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Feedforward Controller Design by Eigenvalue Assignment

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Active feedforward control algorithms have proven to be successful in the attenuation of stationary disturbances such as single frequency, multiple frequencies, and random inputs. In feedforward control the mean square value of a measurable response quantity induced by the disturbance input is minimized by coherent "secondary" control inputs. The traditional design procedure is to choose the location/number of control forces and error sensors based on a physical understanding of the response of the uncontrolled and controlled systems. Thus, an analytical procedure for actuator and sensor design in feedforward controlled systems is nonexistent in contrast to feedback control where pole allocation, optimal control, etc., are extensively used. Recent analytical work has demonstrated that feedforward controlled systems have new eigenproperties. The controlled eigenvalues and eigenfunctions are only a function of the control input location and the error quantity to be minimized. Here, an eigenvalue assignment design methodology for feedforward controllers is presented, and it is based on designing an optimal error sensor such that the controlled system eigenvalues have predetermined values.

Introduction

IN many practical situations, passive methods to attenuate structural vibration have been ineffective. The performance of energy dissipation devices, such as dampers and surface treatments, quickly deteriorates at low frequencies. Changes in the dynamic properties of the system through structural modifications could result in an unacceptable increase in the system's weight. For these types of applications, active methods have shown the capability of increasing the damping or changing the characteristics of the structure without the drawbacks of passive techniques. As a result of advances in digital signal processing, the numbers of practical implementations of active approaches has increased dramatically in recent years.

Feedback and feedforward approaches are the most common techniques employed in the active control of structural vibration as well as their radiated sound fields. Feedback control was initially applied in the field of large space structures, and it has been the subject of intensive research for the last two decades.[1] On the other hand, feedforward was first used in the active control of sound, and it has been in the literature for a much shorter period.[2-5] Though feedforward is the control technique used in this work, a brief description of this method as compared to feedback is presented for the readers not familiar with feedforward control. To this end, Figs. 1a and 1b show schematics of both methods for a single-input, single-output (SISO) control system. The main distinguishing feature is that in feedforward control (Fig. 1b) fully "coherent" information from the disturbance input $F(t)$ is "fed forward" through a compensator G_c to generate the control input $U(t)$. In contrast, in feedback control (Fig. 1a) measured plant outputs are used to predict the states of the system that in turn are input into the compensator to generate the control input. The straightforward implication of this is that feedforward can be selectively tailored to control the response associated to coherent disturbance inputs, nonetheless leaving unaffected the response associated with other incoherent inputs. This behavior should be contrasted to that of feedback where the system characteristics are generally changed for all disturbance inputs. Another distinction between these two approaches is that whereas feedforward is mainly used for stationary disturbances, feedback has the inherent capability of attenuating both stationary and transient disturbances. Though it is not considered here, broadband feedforward systems are capable of controlling transient disturbances. In feedforward control, the compensator is obtained by minimizing a quadratic cost function of a measurable response(s) of the system. The aftermath of this is that an accurate system identification of the control path is not required. On the other hand, a precise modeling of the plant for accurate estimation of the system states is generally critical in feedback approaches.

The design of feedforward controllers is traditionally defined by the selection of the number and location of actuators and sensors based on a physical understanding of the behavior of the uncontrolled and controlled systems. This empirical design approach contrasts with the vast number of analytical tools available to the design of feedback controllers. Techniques such

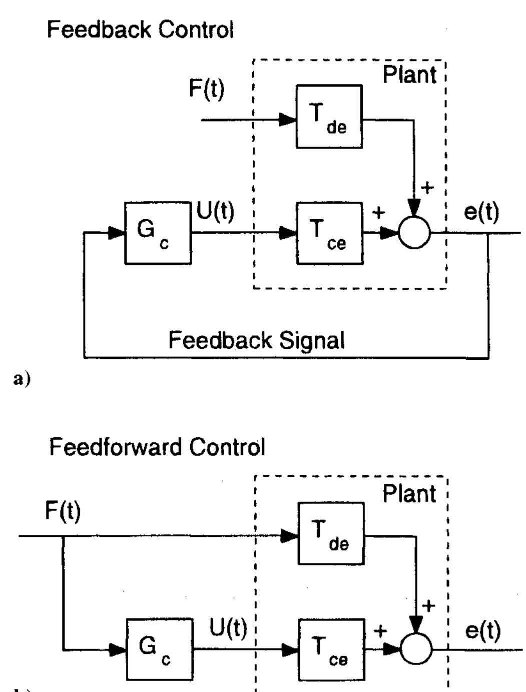

Fig. 1 Schematic of feedback and feedforward SISO control configuration.

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as pole allocation, various state space design methods, optimal control, etc., are extensively used in feedback control.[6] The lack of mathematically based design formulations in feedforward control stems from the small amount of research carried out on understanding the behavior of feedforward controlled systems. Previous research has shown that the controller often modifies the input impedance of the structure to the disturbance resulting in less power transmitted into the system.[7] Recent analytical work has demonstrated that this impedance change is a result of the feedforward controlled structures having new eigenvalues and eigenvectors.[8] Consequently, feedforward control can be used to achieve "active dynamic modification" of the structure to cause the system to behave in a required manner. That is, given a desired set of modal properties, a feedforward controller can then be designed to force the controlled system to have those characteristics. Here, a design formulation is developed for a SISO feedforward controller to actively modify the dynamics of a structure. The design approach is based on an eigenvalue assignment concept where the eigenvalues of the modified or controlled structure are defined in advance and the controller is then designed to meet these specifications. A numerical example is presented and the methodology is also experimentally verified.

Equations of Motion

The structural system is assumed to be an undamped, linear, time invariant, distributed parameter system. Thus, the equations of motion can be written in the form of a partial differential equation as

$$L[W(D,t)] + m(D)\frac{\delta^2 W(D,t)}{\delta t^2} = f(D)F(t) + u(D)U(t) \quad (1)$$

where $W(D, t)$ is the displacement in the domain D, $L[\cdot]$ a self-adjoint linear differential operator, and $m(D)$ the mass distribution. The disturbance or "primary" input is defined by the time-dependent amplitude $F(t)$ and its spatial distribution $f(D)$. The system is controlled by a coherent "secondary" control input defined by the amplitude $U(t)$ and its spatial distribution $u(D)$. As mentioned in the Introduction, feedforward control approaches are used in applications where the disturbance excitation is stationary, i.e., single and multiple frequencies as well as random inputs. Then, the disturbance input $F(t)$ is assumed stationary, and thus $W(D, t)$ and $U(t)$ are also stationary variables. By taking the Fourier transform, the analysis can be carried out on the frequency domain. That is,

$$L[W(D,\omega)] - \omega^2 m(D)\, W(D,\omega) = f(D)\, F(\omega) + u(D)\, U(\omega) \quad (2)$$

Solving for the eigenvalues and eigenfunctions of the homogeneous part of Eq. (2), the response of the system can be written in terms of the mode shape functions by using the expansion theorem as

$$W(D,\omega) = \sum_{n=1}^{N} q_n(\omega)\, \phi_n(D) \quad (3)$$

where $q_n(\omega)$ is the nth generalized coordinate, $\phi_n(D)$ the nth eigenfunction, and N the number of modes included in the analysis. The eigenfunctions satisfy the orthogonality conditions

$$\int_D \phi_n(D)\, m(D)\, \phi_m(D)\, \mathrm{d}D = \delta_{nm} \quad (4a)$$

$$\int_D \phi_n(D) L[\phi_m(D)]\, \mathrm{d}D = \delta_{nm}\omega_n^2 \quad (4b)$$

where δ_{mm} is the Kronecker delta function, and ω_n is the nth natural frequency associated to the eigenfunction $\phi_n(D)$.

The generalized coordinate $q_n(\omega)$ is obtained by replacing Eq. (3) into Eq. (2), premultiplying by $\phi_m(D)$, and using the orthogonality conditions of Eq. (4). Then,

$$q_n(\omega) = [f_n F(\omega) + u_n\, U(\omega)]\, H_n(\omega) \quad (5)$$

where $H_n(\omega) = (\omega_n^2 - \omega^2 + j2\beta_n\omega_n\omega)^{-1}$ is the nth modal frequency response function; j is the imaginary number; β_n is the nth modal damping ratio included here to bound the response at resonance; and f_n and u_n are the nth modal disturbance and control forces defined by the inner products

$$f_n = \int_D \phi_n(D)\, f(D)\, \mathrm{d}D \quad (6a)$$

$$u_n = \int_D \phi_n(D)\, u(D)\, \mathrm{d}D \quad (6b)$$

In the SISO feedforward control system considered here, the optimum frequency component of the control input stems from driving to zero a response of the system that is referred to here as the error variable. The error variable can also be represented as the linear contribution of each modal response as follows:

$$e(\omega) = \sum_{n=1}^{N} q_n(\omega)\, \xi_n \quad (7)$$

where ξ_n is the nth modal component of the error variable that is a function of the physical characteristics of the error transducer to be implemented. The modal error component indicates the relative importance assigned to each mode.

Replacing Eq. (5) in Eq. (7), the error signal can then be set to zero to solve for the optimum control input in terms of the modal quantities. That is,

$$U(\omega) = \left[-\sum_{n=1}^{N} \xi_n f_n H_n(\omega) \Big/ \sum_{n=1}^{N} \xi_n u_n H_n(\omega)\right] F(\omega) = G(\omega)\, F(\omega) \quad (8)$$

where $G(\omega)$ is the feedforward compensator. In this analysis the control system is assumed to be causal, and thus the compensator $G(\omega)$ is realizable.[9] For noncausal control systems, the ideal compensator is not implementable. However, the approach presented here can still be used as a design tool. For single frequency and multiple frequencies disturbances, where causality is not an issue, the proposed design formulation is also applicable.

The feedforward compensator $G(\omega)$ is the transfer function that relates the control input $U(\omega)$ to the disturbance input $F(\omega)$. The controller or compensator $G(\omega)$ in Eq. (8) is defined as the ratio of two transfer functions, where the numerator is the transfer function between the disturbance or primary input and the error variable $T_{de}(\omega)$, whereas the denominator is the transfer function between the secondary control input and the error variable $T_{ce}(\omega)$. The controller $G(\omega)$ can provide valuable information as to the performance of the control system. For example, the poles of $G(\omega)$ would indicate the frequencies at which the control effort would be unbounded if no damping was present in the system.

The traditional view of feedforward control techniques is of "active cancellation" where the uncontrolled modes excited by the primary input are canceled by a secondary input of appropriate magnitude and phase driving the same uncontrolled modes. This view arises from the fact that the system response can be obtained by superimposing the response of the disturbance and control inputs as suggested by Eq. (2). The unwanted consequence of this view is that the design of feedforward controllers has become an empirical technique based largely on a physical understanding of the uncontrolled system. Recent work has shown that the mechanisms for acoustic control, for example, with stationary disturbances is that, in general, the active source modifies the radiation impedance of the disturbance source, thus leading to less power radiated.[10] For active vibration control (AVC) as well as active structural acoustic control (ASAC), the authors have recently demonstrated that feedforward controlled systems have new eigenvalues and eigenfunctions as in feedback control.[8,10] The proposed eigenvalue assignment design procedure is based on this eigenanalysis.[8] For the sake of brevity, only the main concepts of the controlled system eigenanalysis are presented here; a complete description of this formulation can be found in Ref. 8.

For this analysis the controlled system is assumed to be undamped. The dynamic behavior of the controlled system is governed by the characteristics of the controller $G(\omega)$. Multiplying and dividing Eq. (8) by the product of the modal frequency response functions $H_n(\omega)$, $G(\omega)$ can be written as the ratio of two polynomials as follows[8]:

$$G(\omega) = -N(\omega)/P(\omega) \tag{9}$$

with

$$P(\omega) = \sum_{n=1}^{N} \xi_n u_n \prod_{\substack{m=1 \\ m\neq n}}^{N} (\omega_m^2 - \omega^2) \tag{10}$$

The polynomial $P(\omega)$ in the denominator can be shown to be the characteristic polynomial of the controlled system. Thus, the eigenvalues of the controlled system ω_{cl}^2 are obtained by solving for the roots of $P(\omega)$, $P(\omega_{cl}) = 0$. It is not difficult to show that $P(\omega)$ is the numerator of the transfer function $T_{ce}(\omega)$, and thus the controlled system eigenvalues are the zeros of $T_{ce}(\omega)$ (Ref. 8). This implies that we are canceling the zeros of $T_{ce}(\omega)$ with the poles of $G(\omega)$. The order of the polynomial in Eq. (10) is $(N-1)$ in the variable ω^2, and thus the controlled system has $(N-1)$ new eigenvalues. The controller has reduced the dynamic degrees of freedom of the system by one through the constraint imposed on the structure by driving the error signal to zero. The controlled system eigenfunction $\phi_{cl}(D)$ associated with the natural frequency ω_{cl} is easily computed once the controlled system eigenvalues have been determined. They are obtained as a linear combination of the uncontrolled modes, since they have been used as an expansion basis. Then,

$$\phi_{cl}(D) = \sum_{n=1}^{N} \Gamma_{ln}\phi_n(D) \tag{11}$$

where the expansion coefficients are

$$\Gamma_{nl} = C_l\,[u_n/(\omega_{cl}^2 - \omega_n^2)] \tag{12}$$

The constant C_1 in Eq. (12) is included since the controlled mode shapes are arbitrary to a constant multiplier, and it can be computed by requiring $\Sigma_n(\Gamma_{nl})^2 = 1$. It is worthwhile mentioning that Eq. (12) is identical to the relationship found in the computation of the modified eigenfunctions in dynamic local modification techniques.[11] Thus, feedforward control can also be used to actively modify the dynamics of a structure without the drawback of the unwanted increase in system mass generally associated with structural modification methods.[12] Another important observation is that from Eqs. (11) and (12) it can be observed that the eigenproperties of the controlled system are a function of only the control input through u_n and of the error variable to be minimized through the modal error components ξ_n, and they are independent of the disturbance input. This conclusions are again similar to those as found in feedback controlled systems.

Eigenvalue Assignment Design Approach

The theory in the previous section shows that a system under feedforward control has different modal properties from the uncontrolled system with respect to the excitation $F(\omega)$. This implies that by proper control design a structure could be actively modified to behave with new dynamic properties as required by the analyst. The design of feedback control systems is related to the computation of the feedback gains, where techniques such as eigenvalue assignment, eigenvector assignment, linear optimal control, and others are extensively used to this end. On the other hand, the general design of a SISO feedforward control system involves the selection of the control input distribution determined by the modal components u_n and of the error quantity to be minimized. In contrast to feedback control where a substantial number of mathematical design formulations are available, analytical feedforward design approaches are basically nonexistent in the literature. In the following, a feedforward control design procedure based on the eigenvalue assignment technique will be presented. Here, it is assumed the control load is fixed, and thus the modal control inputs u_n are known. The design approach will then focus on the selection of the proper error sensor, whose output will be minimized, to drive the closed loop behavior as required.

In the eigenvalue assignment technique, the controlled system eigenvalues are selected in advance, and then the error sensor is designed so as to match the desired controlled system poles. This implies that the $(N-1)$ controlled eigenvalues ω_{cl}^2 are known quantities and the modal components of the error variable ξ_n are to be computed. Substituting the controlled system eigenvalues into the characteristic polynomial in Eq. (10), we can write

$$P(\omega_{cl}) = \sum_{n=1}^{N} \xi_n u_n \prod_{\substack{m=1 \\ m\neq n}}^{N} (\omega_m^2 - \omega_{cl}^2) = 0 \qquad l = 1, \ldots, (N-1) \tag{13}$$

which can also be written in matrix form as follows:

$$\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{(N-1)1} & S_{(N-1)2} & \cdots & S_{(N-1)N} \end{bmatrix} \begin{Bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_N \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \tag{14}$$

where

$$S_{ln} = u_n \prod_{\substack{m=1 \\ m\neq n}}^{N} (\omega_m^2 - \omega_{cl}^2) \tag{15}$$

The dimension of the matrix in Eq. (14) is $(N-1) \times N$. Careful observation of Eq. (15) shows that if one of the controlled eigenvalues is identical to one of the uncontrolled system eigenvalues, i.e., $\omega_s = \omega_{cr}$ only the term S_{sr} in the sth row in the matrix does not vanish. Therefore, to satisfy the sth equation ξ_s must also be zero. This implies that the sth eigenfunction is unobservable by the error sensor, and thus it can not be affected by the control input. The linear system in Eq. (14) is such that any multiple of the vector $\{\xi_1, \xi_2, \ldots, \xi_N\}^T$ is a solution, and therefore the only relevant information is the relative value between the modal error components. Assuming the Nth mode is observable, ξ_N is set to unity, and by suitably partitioning the matrices in Eq. (14), the modal components can be obtained by solving the reduced linear system of equations

$$\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1(N-1)} \\ S_{21} & S_{22} & \cdots & S_{2(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ S_{(N-1)1} & S_{(N-1)2} & \cdots & S_{(N-1)(N-1)} \end{bmatrix} \begin{Bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_{N-1} \end{Bmatrix} = - \begin{Bmatrix} S_{1N} \\ S_{2N} \\ \vdots \\ S_{(N-1)N} \end{Bmatrix} \tag{16}$$

The vector $\{\xi_1, \xi_2, \ldots, \xi_{N-1}, 1\}^T$ defines the error sensor in the modal domain. Thus, this modal information needs to be transformed into a physical sensor on the structure, and to this end two basic approaches can be implemented. One method utilizes an array of discrete point sensors, i.e., accelerometers, strain gauges, etc. The error variable is obtained by summing the weighted discrete sensor signals. Assuming N_s discrete point displacement sensors, the modal error component becomes

$$\xi_n = \int_D \sum_{i=1}^{N_s} b_i \, \delta(D_i - D) \, \phi_n(D) \, dD \tag{17}$$

where b_i is the weight of the ith point sensor located at D_i and $\delta(D_i - D)$ is the Dirac delta function. Then, Eq. (17) reduces to

$$\xi_n = \sum_{i=1}^{N_s} b_i \, \phi_n(D_i), \qquad n = 1, \ldots, N \tag{18}$$

Equation (18) can also be written in matrix form as

$$[A]\,\{b\} = \{\xi\} \tag{19}$$

where the components of matrix $[A]$, which relates the error quantity in the physical domain to the modal domain, are $A_{ni} = \phi_n(D_i)$, and $\{b\} = \{b_1, b_2, \ldots, b_N\}^T$ is the weighting vector. Assuming $N \geq N_s$ the solution of Eq. (19) can be obtained by the pseudoinverse technique.[13] That is,

$$\{b\} = ([A^T]\,[A])^{-1}\,[A]^T\,\{\xi\} \tag{20}$$

The sum of the output of N_s discrete point sensors is a design approach applicable to any complex structure. However, implementation of this approach could result in considerable signal conditioning effort, particularly in "real-time" digital control where all calculation must be completed in one sampling period. The use of distributed sensors has recently gained acceptance in the control community for their inherent built-in filtering capability of the system response. The flexibility, light weight, and toughness properties of polyvinylidene fluoride polymer (PVDF) films have found application as distributed structural sensors in active control.[14–18] The PVDF film sensor is mounted on the surface of the structure and yields a response proportional to the integral of the strain over the surface of application. In particular, one-dimensional problems are ideally suited for the use of PVDF distributed sensor because any desirable weighted response can be obtained. The design of a physical PVDF sensor from the modal error components ξ_n will be presented for a beam problem. Thus, the domain D becomes the x coordinate in the following derivation.

Since PVDF film is a strain sensor, the modal error component becomes[15]

$$\xi_n = -\alpha h \int_0^L b(x) \, \frac{d^2\phi_n\, x)}{dx^2} \, dx \tag{21}$$

where h is the distance of the upper beam surface to the neutral axis here assumed constant for the sake of clarity in the presentation; α is a constant of proportionality that relates the film electrical and mechanical properties to the surface strain; and $b(x)$ is the function that weights the strain along the beam, and it is determined by the PVDF film's geometry. This function dictates the width of the film and can take either positive or negative values. The negative effective width can be implemented by segmenting the film with out-of-phase wiring of the negative segments, i.e., inverting the polarity of the sensors. To take advantage of the orthogonality conditions of the modes, the weight function $b(x)$ is expanded as follows:

$$b(x) = \sum_{m=1}^{N} b_m \, EI(x) \, \frac{d^2\phi_m(x)}{dx^2} \tag{22}$$

To solve for the unknown expansion coefficients b_m, Eq. (22) is replaced in Eq. (21) and using the orthogonality condition of Eq. (4b) gives

$$b_m = -\,(\xi_m / \alpha h \omega_m^2) \tag{23}$$

That yields the weight function in term of the computed modal error components as follows:

$$b(x) = \sum_{n=1}^{N} \frac{\xi_n}{\omega_n^2} \, EI(x) \, \frac{d^2\phi_n(x)}{dx^2} \tag{24}$$

In this design formulation the control input was assumed fixed, and the error sensor is then configured to achieve the sought controlled system characteristics. However, inspection of the characteristic polynomial in Eq. (13) reveals that the design process could have also been stated as: assuming the error sensor fixed, the control input distribution is then designed to obtain the desired poles. This is simply accomplished by replacing ξ_n by u_n and vice versa in Eqs. (13–24). These two eigenvalue assignment design concepts differ only in that the associated controlled eigenfunctions are different since the expansion coefficients Γ_{ln} in Eq. (12) are determined by u_n. Greater design flexibility is possible if both the control load distribution and the weighted error variable are designed simultaneously. In this case, the controlled system eigenvalues as well as the associated eigenfunctions could be preselected in advance using an eigenvalue-eigenfunction assignment technique. The main application of this approach would be in active structural acoustic control where the radiation efficiency of the controlled mode shapes are as important as the resonance conditions.[19]

Numerical Example

The applicability of the design formulation presented here is demonstrated for the uniform simply supported beam of Fig. 2. The beam is made of steel and has bending stiffness EI = 93 N·m², mass per unit length m = 2.181 N·s²/m², and beam length L = 0.38 m. To compute the response of the system, a modal damping ratio of 0.1% is assumed in all modes (β_n = 0.001), and only the first four modes are included in the analysis. The beam is excited by a concentrated force located at x_d = 0.1L, and the response is controlled by another point force

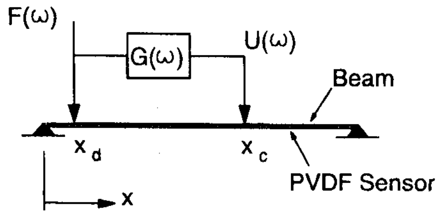

Fig. 2 Simply supported beam example problem.

placed at $x_c = 0.65L$. In this example, the error sensor, whose output is minimized, is a distributed PVDF shaped film mounted on the beam surface. This sensor is designed to induce the desired dynamic characteristics of the structure. Since the proposed design formulation is based on the modal representation of the system's response, the eigenproperties of the uncontrolled beam to be used in the analysis are given by

$$\omega_n = \left(\frac{n\pi}{L}\right)^2 \sqrt{\frac{EI}{m}} \tag{25a}$$

$$\phi_n(x) = \sqrt{\frac{2}{mL}} \sin\left(\frac{n\pi x}{L}\right) \tag{25b}$$

In this example problem, we seek to arbitrarily shift the natural frequencies of the system in the frequency range of the first four modes. As mentioned before, the controlled system has one less dynamic degree of freedom due to the constraint imposed in the system as result of driving the error signal to zero. Thus, the number of controlled modes is three, and the desired beam-control system natural frequencies ω_{cl} are shown in Table 1. The uncontrolled natural frequencies are also shown in the same table. These desired natural frequencies are replaced into Eq. (15) to compute the coefficients of the matrix and independent vector in Eq. (16). Solving this linear system of equations for the modal error components ξ_n defines the error sensor in the modal space. To convert this into a PVDF distributed sensor, the modal components ξ_n are replaced into Eq. (24) where the weight function $b(x)$ is computed. This function determines the width profile of the PVDF sensor, which is shown in Fig. 3.

To illustrate the dynamic behavior of the beam before and after control, the acceleration response at the input force location was computed. The acceleration response is obtained by multiplying Eq. (3) by ω^2. Assuming a unit amplitude of the input force, the magnitude and phase of the acceleration at x_d are shown in Fig. 4 as function of the frequency ω. The dashed line is the analytical response of the uncontrolled beam obtained by using Eqs. (3) and (5) and setting $U(\omega)$ to zero. The continuous line corresponds to the controlled system calculated from Eqs. (3), (5), and (8) and shows resonance behavior at the preselected frequencies ω_{cl}. Thus, on driving the weighted response output from the PVDF sensor the beam-control system has the sought eigenvalues. The associated controlled eigenfunctions, computed using Eqs. (11) and (12), are shown in Fig. 5.

Table 1 Uncontrolled and controlled system natural frequencies

	Natural frequency, Hz			
	Uncontrolled, $f_n = \omega_n/2\pi$		Controlled, $f_{cl} = \omega_{cl}/2\pi$	
Mode	Analysis	Experimental	Analysis	Experimental
1	71.0	75	425	424
2	284.1	278	780	750
3	639.3	626	1000	1030
4	1136.5	1120		
5	1775.8	1450		

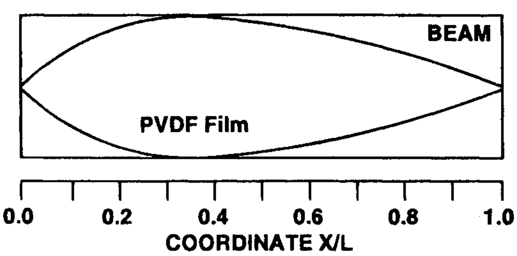

Fig. 3 PVDF film sensor geometry.

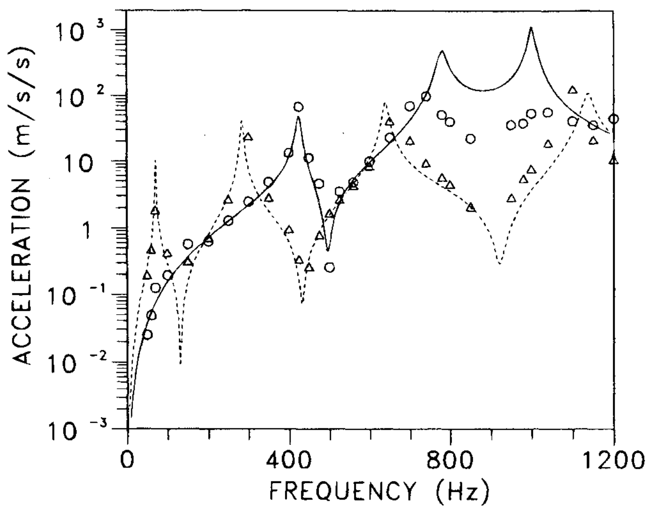

Fig. 4 Acceleration response at disturbance force location: - - - - uncontrolled analytical, △ uncontrolled experimental, ——controlled analytical, ○ controlled experimental.

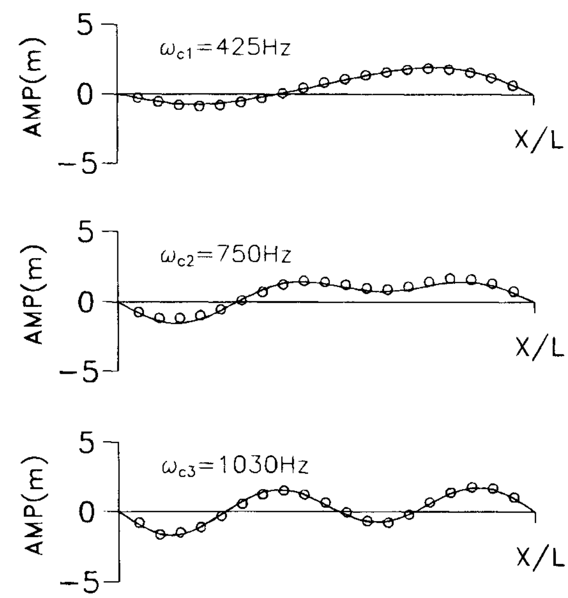

Fig. 5 Controlled system eigenfunctions: ——analytical, ○ experimental.

The numerical example was also experimentally verified using the setup shown in Fig. 6. The experimental rig consists of a steel beam mounted in a heavy rigid frame by shims attached to the edges of the beam to simulate the simply supported boundary conditions. Other beam parameters are identical to the numerical example. The disturbance and control forces were applied by two Ling minishakers attached pointwise to the beam and suspended by elastic cords to minimize their coupling with the structure. Kistler force transducers were used to monitor the shaker forces. A rectangular PVDF film was attached to the beam; and the desired shape, calculated using the theory of the previous section, was obtained by etching the surface electrode with etchant. The shape of the resultant sensor for this example is shown in Fig. 3. The output signal from the PVDF is the error signal that the control force will be required to cancel. Two Brüel and Kjær miniaccelerometers were also attached to the beam. One was used to measure the beam response at the disturbance force location, whereas the other is movable and was used to measure mode shapes. The natural frequencies of the experimental setup configuration

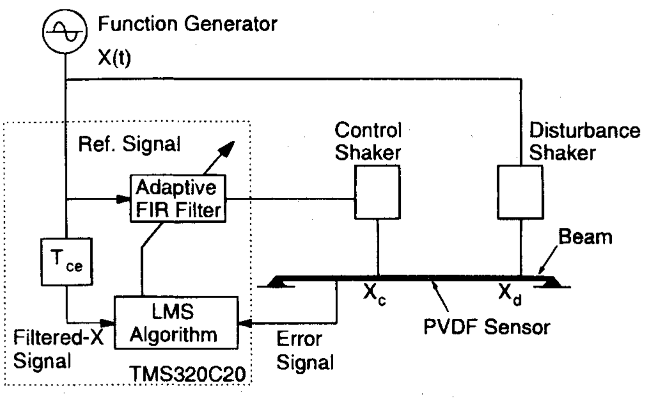

Fig. 6 Time domain filtered-x LMS control block diagram.

were measured, and they are compared to the computed natural frequencies, Eq. (25a), in Table 1 for the first five modes. Good agreement for the first four resonances is observed, although the results show a difference of 22% for the fifth mode. This arrangement will affect the results of the experiment somewhat but the concept of the formulation is still clearly demonstrated.

To experimentally verify the previous analytical results, a narrowband adaptive feedforward control system based upon the "filtered-x" version of the least mean square (LMS) algorithm was implemented.[20] The block diagram of this control system is given in Fig. 6. The algorithm was coded in assembly language and downloaded to a PC-based TMS320C20 digital signal processor. As can be seen in Fig. 6, the disturbance signal x_k is filtered by the transfer function between the control input and error output, T_{ce}. This signal in conjunction with the error signal is used by the LMS algorithm to adapt the coefficients of a finite impulse response (FIR) adaptive filter. The system was excited with a sinusoidal disturbance, and thus only two coefficients were used for T_{ce} and the adaptive filter. The reference signal was directly taken from the function generator, thus providing a coherent control signal with the disturbance. As described in Ref. 20, the adaptive LMS algorithm minimizes the mean square value of the PVDF error signal. Thus, on convergence of the controller, its optimal transfer function would be identical to the analytically predicted in Eq. (8) at the excitation frequency.

The experiment was carried out for a set of frequencies in the range 50–1200 Hz. The analytical and experimental results are plotted in the same figures. The acceleration at the input force location, monitored by one of the Brüel and Kjær miniaccelerometers, before and after control is plotted in Fig. 4 with symbols. The magnitude of the analytical and experimental controlled response shows some discrepancy as the frequency increases. This is due to the fact that the simply supported beam model does not accurately represent the experimental setup after the fourth mode as shown in Table 1. However, it is apparent from Fig. 4 that the experimental resonance conditions of the modified system occur at nearly the same frequencies as predicted by the analysis. The controlled system eigenfunctions were also experimentally determined by driving the controlled system at the frequency of the resonance peaks and measuring the beam response at equally spaced axial positions with the moveable miniaccelerometer. The measured mode shapes, after being normalized with respect to the mass distribution, are plotted in Fig. 5 with symbols. Excellent agreement with the computed mode shapes is demonstrated. Note that the computed mode shapes were calculated from the individual eigenfunction expression in Eqs. (11) and (12), rather than the total response of the system at the resonance points. The results of the experiment thus validates the analytical formulation presented in this work.

Conclusions

A design formulation based on the eigenvalue assignment technique is developed for designing feedforward controllers. The formulation can be stated as follows: the controlled system eigenvalues are selected in advance. An optimum error sensor is then designed so as to cause the system to match the desired controlled system poles when control is applied. This analytical design approach clearly contrasts with the traditional methods of selecting the location/number of control forces and error sensors based simply on a physical understanding of the uncontrolled system response. The proposed design approach is a natural progression of recent analytical work that demonstrated that feedforward controlled systems have new eigenproperties and is the basis of more sophisticated approaches. A numerical example is presented to demonstrate the proposed technique. In addition, the control algorithm was implemented on a signal processing board, and experimental data was obtained and compared to the previous derived analytical results. Good agreement between analytical predictions and experimental measurement was found.

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